SB^{3}C2016
Summer Biomechanics, Bioengineering and Biotransport Conference
June 29 –July 2, National Harbor, MD, USA

NUMERICAL CONSTRAINT FOR TRACKING TAGGED MAGNETIC RESONANCE IMAGES IN BIOMECHANICAL SIMULATIONS

Arnold David Gomez (1), Chan Deva (2), Yuan-Chiao Lu (2), Dzung Pham (2), Philip Bayly (3), Jerry Prince (1)

(1) Electrical and Computer Engineering Dept.
Johns Hopkins University
Baltimore, MD, USA

(2) Center for Neuroscience and Regenerative Medicine, Henry Jackson Foundation,
Bethesda, MD, USA

(3) Mechanical Engineering Department
Washington University in St. Louis
St. Louis, MO, USA

INTRODUCTION

Tissue deformation is plays a fundamental role in many vital organs, including cardiac function, and displacement during impact leading to traumatic brain injury [1-2]. Motion estimation using tagged MRI has had an expanding role as a functional biomarker in clinical practice, and as method for providing experimental validation of mechanical simulations [1,3]. Harmonic phase (HARP) analysis is a popular approach to track tagged images by extracting phase vector images, which effectively become conserved material properties associated to a reference configuration i.e., values that travel with the tissue as it deforms [4]. However, generation 3D displacement fields requires volumetric data acquired over multiple repetitions, which, given experimental inconsistency, magnify the effects of motion, partial volume, and other artifacts. To reduce these effects, estimation of 3D is often accompanied by regularization and filtering, but these strategies often elicit additional parameters that are difficult or impossible to measure experimentally, and thus may induce additional uncertainty and error to the estimation.

This study describes the HARP tracking process as enforcing conservation of an additional material property—the phase vector at each material location. This novel description enables embedding the tracking process into finite element (FE) simulations, and including mechanical features inherent to numerical biomechanical analysis such as realistic boundary conditions, and material properties. Because these features can be measured or inferred experimentally, FE-based HARP tracking (HARP-FE) produces accurate and representative 3D strain fields. To illustrate these ideas, HARP-FE was derived and deployed to: (1) calculate the principal stress distribution of a pre-strained incompressible tissue sample using synthetic image data, and (2) estimate motion of a brain phantom subject to rotational acceleration using experimental MRI data.

METHODS

Theory—A harmonic phase vector field, $\boldsymbol{\phi}$, is typically extracted from complex MRI by applying a band-pass filter centered at the tagging frequency [4]. HARP tracking consists of finding spatial coordinates, $\boldsymbol{x}$, with the same phase values as those in a set of initial material points, $\boldsymbol{X}$, i.e., $\boldsymbol{x}(\boldsymbol{X})$ such that $\boldsymbol{\phi}(\boldsymbol{X}, t_0) = \boldsymbol{\phi}(\boldsymbol{x}(\boldsymbol{X}), t)$. This relationship is equivalent to a mass transport expression stemming from the weak formulation of conservation of the phase density field $\boldsymbol{\phi}$ in a control volume [5]. In a mass-conservative field, it can be shown that

$$\frac{\mathrm{D}(\boldsymbol{X}, t_0)}{\mathrm{D}t} = \frac{\partial \boldsymbol{\phi}(\boldsymbol{x}, t)}{\partial t} + \nabla \boldsymbol{\phi}(\boldsymbol{x}, t) \cdot \boldsymbol{v}(\boldsymbol{x}, t) = 0, \tag{1}$$

where $\boldsymbol{v}$ is velocity. The spatial terms above represent a phase version of the familiar *optical flow* equation used for motion estimation via image registration [6]. With the Lagrange multiplier, λ, the material term yields the HARP-FE tracking constraint,

$$\boldsymbol{f}_{harp} = \lambda(t)[\boldsymbol{\phi}(\boldsymbol{x}(\boldsymbol{X}), t) - \boldsymbol{\phi}(\boldsymbol{X}, t_0)], \tag{2}$$

which is directly applied (as an additional phase-based body force) to the virtual work equation from conservation of momentum, or

$$\delta \mathrm{W} = \int_{\mathcal{R}} \boldsymbol{P} : \delta \dot{\boldsymbol{F}}\, dV - \int_{\mathcal{R}} \boldsymbol{f} \cdot \delta \boldsymbol{v}\, dV + \oint_{\mathcal{S}} \boldsymbol{t} \cdot \delta \boldsymbol{v}\, dV dS = 0, \tag{3}$$

where $\boldsymbol{P}$ is the material-dependent 2nd Piola-Kirchhoff stress tensor, $\boldsymbol{F}$ is the deformation gradient, $\boldsymbol{t}$ represents external loads, and $\boldsymbol{f}$ represents all internal forces, including tracking $\boldsymbol{f} = \boldsymbol{f}_o + \boldsymbol{f}_{harp}$. The HARP-FE constraint was implemented as a plug-in extension for the FEBio Software Suite [7] using (2), and tangent stiffness contribution defined by the linearization of (3), i.e,

$$\mathrm{D}\delta\mathrm{W}_{harp}(\boldsymbol{x},\delta\boldsymbol{v})[\mathrm{u}] = \int_{\mathcal{R}} \lambda(t)\nabla\boldsymbol{\phi}(\boldsymbol{X},t_0)\,\delta\boldsymbol{v}\cdot\delta\boldsymbol{u}\,dV, \qquad (4)$$

where $\mathrm{D}f(\mathrm{x})[\mathrm{u}]$ is the directional (or Gateaux) derivative.

Approximation of Stress Distribution—The goal this experiment was to demonstrate compatibility and benefits of including mechanical analysis along with motion estimation. An incompressible tissue sample was modeled as a rectangular, pre-stretched (10% in the x-direction), Veronda-Westmann solid (C_1= 0.5 kPa, C_2= 2.5), subject to force at its center (Fig. 1a). Synthetic tagged images in each direction (64×64×2 px, 0.2×0.2mm, complementary spatial modulation of magnetization, or C-SPAMM, 12 px spacing) were generated before and after the application of the force for the reference and deformed configurations. These images were decomposed into magnitude and phase to define $\boldsymbol{\phi}$ [4], and used as input for HARP-FE, which was performed first without any constraints, then with added boundary conditions at the ends, and, finally, with boundary conditions as well as pre-stretching. The multiplier $\lambda(t)$ was set to <0.05 rad mean tracking error. Combined grid images (x and y) appear in Fig. 1b-c.

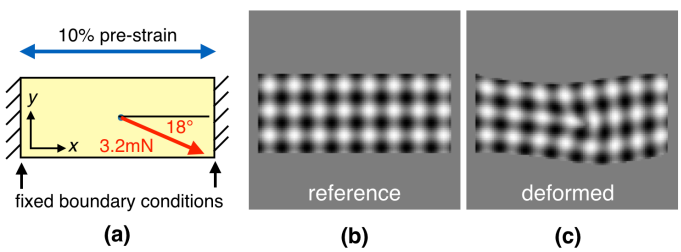

Figure 1: Displacement information from a model (a) was used to create synthetic images (b-c) and stress estimation via HARP-FE.

Experimental Motion Estimation of Brain Phantom—This experiment was designed to evaluate the HARP-FE using actual MRI data against a conventional tracking approach. A gelatin phantom was mounted in a MRI-compatible rotational apparatus (Fig 2a) and subsequently imaged while experiencing sudden angular acceleration (Siemens 3.0T mMR Biograph scanner, 13 axial slices, 160×24 zero padded to 160×160 px, 1.5×1.5mm, 15×15mm, SPAMM sequence, 12 px spacing, see [8] for more information). The images were interpolated into a 160×160×130 volume, and decomposed for tracking as described. Phase images were tracked with conventional 3D-HARP [4] and with HARP-FE, which was performed in the geometry shown in Fig. 2b. $\lambda(t)$ was set to <0.25 rad mean tracking error.

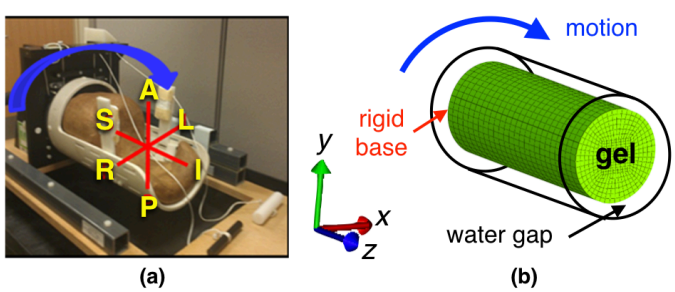

Figure 2: An MRI-compatible rotation apparatus (a) was used to image a gel phantom [8]. The bottom of the phantom was affixed to the container and other surfaces were surrounded by water (b). The gel volume was discretized into a FE domain for tracking. The letters indicate standard imaging directions, and the 3D arrows indicate the FE coordinate system.

RESULTS

Figure 3 shows stress approximations using different variations of HARP-FE. Although material and tracking parameters are constant, the unconstrained solution allows small differences in deformation (labeled with arrows), which have little effect on kinematics, but yield large changes in stress due to the exponential nature of the material. Incorporation of boundary conditions improves the solution, but the most notable differences occur with inclusion of pre-strain.

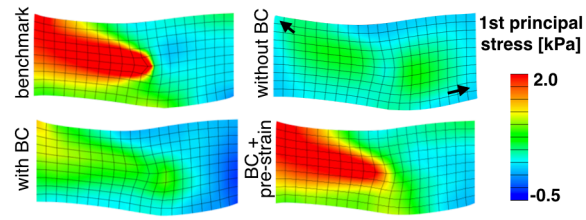

Figure 3: Adding boundary conditions (BC) reduces but does not eliminate visible discrepancies in stress distribution (arrows). Pre-strain with BCs allows excellent agreement with the benchmark.

Figure 4 shows a comparison between conventional 3D-HARP, and HARP-FE. Because the former does not include regularization, it is susceptible to error in areas with imaging artifacts, such as the boundaries, producing intractable deformations. The deformation via HARP-FE is subject to elastic regularization minimizing the artifacts.

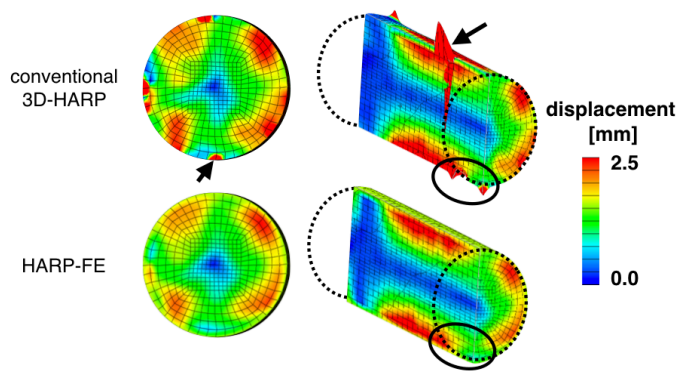

Figure 4: Radial (left), and axial (right) views of magnitude displacement maps show artifacts (arrows and circles) that eliminated via continuum mechanics regularization.

DISCUSSION

Although FE-based HARP tracking requires mesh generation, it offers unique advantages that enable calculation of stress as well as kinematics and exhibits improved tolerance to imaging artifacts. Because it can be performed simultaneously with physical loads, or as a stand-alone driver of deformation, this technique is well suited for different applications, including inverse parameter identification.

ACKNOWLEDGEMENTS

This research was funded with NHI grant R01-NS055951 and support by the Center for Neuroscience and Regenerative Medicine.

REFERENCES

[1] Gotte, M. et al. *J Am Coll Cardiol.* Mar 1;37(3):808-17. 2001.
[2] Coats, et al. NNS Meeting. 2011
[3] Bayly, P. et al. *Annu Rev Biomed Eng*. 14: 369–396. 2012.
[4] Osman, N. et al. Magn Reson Med. Dec; 42(6): 1048–1060. 1999.
[5] Spencer, A. J. M. *Continuum Mechanics.* Dover. 2004.
[6] Horn, B. et al. *Artificial Intelligence* 59 (1993) 81-87.
[7] Maas, S. et al. *J Biomech Eng.* Jan 134(1):011005. 2012.
[8] Knutsen, A. et al. J Biomech. 2014 Nov 7;47(14):3475-81.